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Scheduling in Wireless Ad hoc Networks with Successive Interference Cancellation

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Abstract—Successive interference cancellation (SIC) is an effective way of multipacket reception (MPR) to combat interference at the physical layer. To understand the potential MPR advantages, we study link scheduling in ad hoc networks with SIC at physical layer. The fact that the links detected sequentially by SIC are correlated at the receiver poses key technical challenges. A link can be interfered indirectly when the detecting and removing of the correlated signals fail. We characterize the link dependence and propose simultaneity graph (SG) to capture the effect of SIC. Then interference number is defined to measure the interference of a link and facilitate the design of scheduling scheme. We show that scheduling over SG is NP-hard and the maximum interference number bounds the performance of maximal greedy schemes. An independent set based greedy scheme is explored to efficiently construct maximal feasible schedule. Moreover, with careful selection of link ordering, we present a scheduling scheme that improves the bound. The performance is evaluated by both simulations and measurements in testbed. The throughput gain is on average 40% and up to 180% over IEEE 802.11. The complexity of SG is comparable with that of conflict graph, especially when the network size is not large.

Keywords-Link scheduling; ad hoc network; successive interference cancellation.

I. INTRODUCTION

Modern wireless communication is interference-limited. Due to the broadcast nature, what arrives at the receiver is a composite signal consisting of all near-by transmissions. However, the receiver tries to decode only one transmission by regarding all the others as interference and noise. When the arrivals of multiple transmissions overlap, collision occurs and the reception fails.

There are two major ways to combat the interference. The first is interference avoidance, which is at the high layers to arrange the transmission to avoid the harmful interference. This way is easy to adopt but inherently unable to provide high throughput. Many recent works show that, even with perfect network coordination, the performance is still poor [1]. The second way is to embrace the interference, i.e., all packets in a composite signal are decoded. Such capability of multiple packet reception (MPR) is a significant progress in signal processing. Recently, theoretic analysis [2, 3] has verified the effectiveness of MPR.

Following the second approach, we further ask: with the MPR techniques, do the non-MPR upper-layer protocols work well, or how to coordinate the transmissions? Though significant progress has been made in the MPR techniques at the physical layer, little attention has been paid to the design of support protocols. As not all composite signals are decodable, it is indispensable to avoid harmful collisions (i.e., when the involved signals cannot be separated). In particular, there are specific requirements to ensure the feasibility of an MPR method. It is necessary to coordinate the transmissions carefully to meet the requirements.

We focus on link scheduling in ad hoc networks with successive interference cancellation (SIC) at the physical layer. SIC is a simple but powerful technique to perform MPR. Though scheduling as a classic issue has been extensively studied [4, 5], there are new challenges posed by SIC. In general, before SIC extracts the desired signal, the receiver must detect and remove the signals with greater strength. When detecting the desired signal, only the interfering signals with weaker strength are retained. Therefore, it is necessary to differentiate the removable (i.e., stronger) interference from the remaining. However, in existing works available in literature, the interfering signals are handled as a whole.

The sequential detection results in the dependence among signals and introduces a new type of interference. In particular, the reception of a desired signal relies on the successful detection of the stronger interfering signal(s). Consider three signals, $S_1$, $S_2$ and $S_3$. Suppose, at the intended receiver of $S_1$, the signal strength of $S_2$ is greater than that of $S_1$. As a result, the detection of $S_1$ depends on that of $S_2$. How does $S_3$ affect the detection of $S_1$? It is possible that $S_3$ does not interfere $S_1$ directly. If $S_3$ and $S_1$ overlap and $S_3$ can be detected in the presence of $S_1$, one can remove $S_3$ and further detect $S_1$ successfully. On the other hand, if all three signals exist and $S_3$ can interfere the detection of $S_2$, it is impossible to extract $S_1$ without removing $S_2$. The effect of $S_3$ on $S_1$ is a new type of interference, indirect interference. The major feature is that it does not act directly on the desired signal, but on the correlated signal and eventually propagates to the detection of the desired signal. To facilitate efficient scheduling, both direct
and indirect interference should be taken into account.

To achieve the objective, we make the following contributions. First, we propose a link interference model to describe the link dependence. Second, a new network graph model, simultaneity graph (SG), is introduced. The SG is built on the conflict graph (CG) model [6] with a key new component, super vertex, to capture the impact of SIC. A super vertex contains both a link and its correlated links(s), so that indirect interference can be characterized accurately. Third, to fully capture the effect of both direct and indirect interference, interference number is defined which plays a similar role as node degree in CG to facilitate the design of a scheduling scheme. As scheduling over SG is NP-hard, we resort to an efficient heuristic solution. We show that the maximum interference number bounds the performance of maximal greedy schemes. An independent set based greedy scheme is proposed to efficiently construct maximal feasible schedule. Moreover, with careful selection of link ordering, we present a scheduling scheme that achieves better bound. Finally, the performance is evaluated by simulations. It is shown that the schemes perform very well, e.g., the throughput gain is on average 40% and up to 180% over IEEE 802.11. The overhead of SG is comparable to CG, especially when the network size is not large, which is a typical scenario for time-division multiple access (TDMA) scheduling.

The rest of the paper is organized as follow. Section II overviews the related work and Section III describes the system model and motivation of our work. Section IV presents the new interference model. Section V discusses the scheduling schemes. Sections VI and VII present the results of simulation and measurement in testbed, respectively. Finally, we conclude the research in Section VIII. The proofs of most claims and properties are given in Appendix.

II. RELATED WORK

It is a fundamental issue to handle the interference in wireless networks. Here we summarize only some of the works closely related to ours.

**MPR:** The earliest MPR technique is multi-user detection (MUD) [7], which attempts to extract all involved transmissions from the composite received signal. There are several different ways to perform interference cancellation (IC) [7]: parallel, successive, and hybrid. The parallel IC iteratively detects all signals at once, while the successive IC detects the signals sequentially. The hybrid IC is a combination of the two. Recently, Zigzag decoding is proposed to resolve the collision by using multiple composite signals [8]. Among them, successive interference cancellation (SIC) is the simplest one, whose effectiveness in ad hoc networks has been already verified experimentally [9].

**Wireless network with MPR:** Capacity analysis shows that MPR, or SIC, improves the performance significantly in an ad hoc network [2, 3]. To realize the potential of MPR, network protocols must be designed carefully. There are some studies to support MPR in a centralized network [10, 11] and in a distributed scenario, e.g., distributed MAC [12] and joint routing and scheduling [13].

Scheduling with MPR: Interference-aware scheduling is a classic issue [4–6, 14]. The famous link interference models, e.g., the protocol and physical models [1], are proposed originally for networks with single packet reception. To deal with the MPR, the protocol model is extended by growing the number of permitted interferer from zero to $N$ $(N \geq 1)$ [13], while the physical model is enhanced by allowing reception with a lower SINR threshold [12]. The model used in [10, 11] correlates the reception probability with the number of concurrent transmissions, while neglecting the difference among transmissions.

Network graph is a powerful way to model the network-wide link relations. Most of the existing models (such as CG) fail to capture the impact of SIC. Even though there are many scheduling schemes over CG, most of them [5, 6, 14] are node degree-based and not directly applicable to the newly proposed simultaneity graph.

III. SYSTEM MODEL AND MOTIVATION

Consider a wireless network of $N$ stationary nodes and $n$ links. A link is denoted by $L_i$ or $L_{SR}$ with transmitter $S$ and receiver $R$. Table I summarizes the important notations used in this paper. We assume that: (i) the signal removal of SIC is perfect; (ii) the reception threshold, i.e., required received signal to interference plus noise ratio (SINR), is always larger than one at all nodes; and (iii) each node has an omni-directional antenna, and is not able to transmit multiple packets simultaneously, or to transmit and receive simultaneously. Assumption (iii) was previously stated as primary interference, e.g., in [5].

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$P_S(Y)$</td>
<td>the received signal strength at node $X$ from node $Y$</td>
</tr>
<tr>
<td>$\beta_{ST}$</td>
<td>the reception threshold at $X$ for the signal from $Y$</td>
</tr>
<tr>
<td>LG(SX)</td>
<td>the set of links interferring link $L$ at $X$</td>
</tr>
<tr>
<td>DL(SX)</td>
<td>the set of links on which link $L$ depends at $X$</td>
</tr>
<tr>
<td>SG</td>
<td>simultaneity graph, e.g., $SG=(V,E)$</td>
</tr>
<tr>
<td>SLED</td>
<td>the incident vertex set of link set $LS$</td>
</tr>
<tr>
<td>IVS(LS)</td>
<td>the incident link set of vertex set $VS$</td>
</tr>
<tr>
<td>LVS(VS)</td>
<td>the relevant vertex set of link $L$</td>
</tr>
<tr>
<td>IN(L)</td>
<td>the incoming number of link $L$ in $SG$</td>
</tr>
<tr>
<td>O(L)</td>
<td>the outgoing number of link $L$ in $SG$</td>
</tr>
<tr>
<td>IN(L)</td>
<td>the interference number (IN) of link $L$ in $SG$</td>
</tr>
<tr>
<td>O(L)</td>
<td>the IN difference of link $L$ in $SG$</td>
</tr>
<tr>
<td>S(M)</td>
<td>the maximum outgoing number in $SG$</td>
</tr>
<tr>
<td>S(LM)</td>
<td>the maximum interference number in $SG$</td>
</tr>
</tbody>
</table>

To combat noise, a minimum received power is required to assure signal reception. Let $P_{R}(S)$ denote the received signal power at node $R$ from node $S$ and $P_{N}$ the power of noise. For link $L_{SR}$, detection succeeds if the received signal-to-noise ratio (SNR) satisfies

$$\frac{P_{R}(S)}{P_{N}} \geq \beta_{SR}$$

where $\beta_{SR}$ specifies the reception threshold at node $R$ for the signal from node $S$. As transmission rate and required transmission accuracy can be different among links, $\beta_{SR}$ may not be the same for different transmitter $S$. 

TABLE I: Summary of the main notations
When collision occurs, the desired signal can be correctly detected by exploiting capture effect. Suppose there are transmissions over two links $L_{S,R_1}$ and $L_{S,R_2}$, detection at node $R_1$ of the signal of $L_{S,R_1}$ succeeds if the SINR satisfies
\[ \frac{P_R(S_1)}{P_{No} + P_R(S_2)} \geq \beta_{S,R_1}. \]  
\[ (2) \]

Evidently, as $\beta_{XY} > 1$, when the interfering signal is stronger, capture effect cannot help to extract the desired signal. Interference cancellation techniques can be adopted to resolve the collision. In particular, when SIC is available, $R_1$ can first detect and remove the signal from $S_2$, leaving only the desired signal. The required conditions are given by
\[ \frac{P_R(S_2)}{P_{No} + P_R(S_1)} \geq \beta_{S,R_1} \]  
\[ (3) \]
\[ \frac{P_R(S_1)}{P_{No}} \geq \beta_{S,R_1}. \]  
\[ (4) \]

If there are more than two collided signals, before detecting the desired signal, the receiver node should apply SIC multiple times to remove all interfering signals with greater strength. As a result, the detection of a signal depends on that of all interfering signals with greater strength.

**Motivation:** The fact that the links detected sequentially by SIC are tightly correlated at the receiver poses key technical challenges to extract a desired signal. As illustrated in Fig. 1, in all three cases, all the links (i.e., $L_1, L_2$ and $L_3$) are active. At node $B$, the strength of the desired signal (i.e., from node $A$) is the weakest. Node $B$ should detect and remove the signals of $L_2$ and $L_3$ in turn to obtain the signal of $L_1$. The whole process succeeds in (a), but fails in both (b) and (c).

One should differentiate simultaneous transmissions enabled by capture effect from those by SIC. When (2) holds, whether or not the desired signal can be detected is determined by the capability of the signal against interference. Even with additional interfering signals, the detection can be successful if the overall interference is tolerable. In this case, the reception of desired signal is independent of that of interfering signals. In comparison, when (3) and (4) hold, the detection of desired signal is determined by both the desired and interfering signals. A new signal can interfere either the reception of desired signal, or that of interfering signal. The latter case, in fact, introduces a new type of interference, referred to as indirect interference.

To see how indirect interference occurs, consider Fig. 1(b), where $L_1$ and $L_2$ are permitted to transmit simultaneously. After $L_3$ becomes active, as the signals of $L_3$ and $L_2$ have similar power levels, node $B$ cannot detect the signal of $L_2$ in the presence of $L_3$. Due to the dependence of $L_1$ on $L_2$, without removing the signal of $L_2$, node $B$ eventually fails to detect $L_1$. On the other hand, observe that $L_3$ and $L_1$ can coexist. Node $B$ is able to detect the signal of $L_3$, and then remove the interfering signal, leaving alone the desired signal of $L_1$. The effect of $L_3$ acts not directly on $L_1$, but first on the detection on the correlated signal of $L_2$. The dependence of $L_1$ on $L_2$ makes it possible that the interference of $L_3$ propagates to the detection on $L_1$.

It is necessary to treat the direct and indirect interference differently. To avoid indirect interference, in addition to preventing the transmission of the interfering link, it is feasible to eliminate the link dependence by silencing the correlated link. In both (b) and (c), $L_3$ can interfere $L_1$. In (c), as $L_3$ directly interferes $L_1$, simultaneous transmissions of $L_3$ and $L_1$ are not permitted. However, in (b), the two links can transmit concurrently after $L_2$ is silenced.

**IV. THE INTERFERENCE MODEL**

We model the interference at two levels. A link interference model specifies at a given node the relation among signals of different links, while a network interference model specifies whether a set of links can transmit concurrently.

**A. Link Interference Model**

We start from the famous protocol and physical models [1]. Both of them do not capture the effect of SIC. In the protocol model, $L_{S,R_1}$ is interfered by $L_{S,R_2}$ if
\[ \frac{P_R(S_1)}{P_{No} + P_R(S_2)} < \beta_{S,R_1}. \]  
\[ (5) \]

When there are more than one interfering link, the protocol model describes the interference of each of them on $L_{S,R_1}$. On the other hand, physical model captures the accumulative effect. Denote the interfering links by $L_{S,R_2},..., L_{S,R_I}$ ($I > 2$), $L_{S,R_i}$ is interfered if
\[ \frac{P_R(S_1)}{P_{No} + \sum_{1<i \leq I} P_R(S_i)} < \beta_{S,R_i}. \]  
\[ (6) \]

For simplicity, we extend the protocol model to characterize the link relation when SIC is applied. Consider, at node $X$, there are two links, $L_{S,R_1}$ and $L_{S,R_2}$, and suppose the received signal to noise ratio is larger than the reception threshold (i.e., $P_X(S_i)/(P_{No} \geq \beta_{S,X}, i = 1,2$). Three relations are defined between the two links, where “L−” refers to the locality of link interference (i.e., at node $X$).

- $L_{S,R_i}$ is L-independent of $L_{S,R_j}$ if $L_{S,R_i}$ is detectable in the presence of $L_{S,R_j}$, i.e., $P_X(S_i)/(P_{No} + P_X(S_j)) \geq \beta_{S,X}$.
- $L_{S,R_i}$ is L-dependent on $L_{S,R_j}$ if, in the presence of $L_{S,R_j}$, $L_{S,R_i}$ can be detected and removed, i.e., $P_X(S_i)/(P_{No} + P_X(S_j)) \geq \beta_{S,X}$.
- $L_{S,R_i}$ is L-interfered by $L_{S,R_j}$ if none of them is detectable when they coexist.

The L-interfered relation is symmetric while L-independent and L-dependent relations are not. At a given receiver node, when the reception threshold of $L_1$ is the same as that of $L_2$, Fig. 1: Examples of a three-link network. From (a), (b) is obtained by moving $L_1$ and $L_3$ to the right side and (c) is obtained by moving $L_3$ to the right side.

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When there are more than one interfering link, the protocol model describes the interference of each of them on $L_{S,R_1}$. On the other hand, physical model captures the accumulative effect. Denote the interfering links by $L_{S,R_2},..., L_{S,R_I}$ ($I > 2$), $L_{S,R_i}$ is interfered if
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- $L_{S,R_i}$ is L-dependent on $L_{S,R_j}$ if, in the presence of $L_{S,R_j}$, $L_{S,R_i}$ can be detected and removed, i.e., $P_X(S_i)/(P_{No} + P_X(S_j)) \geq \beta_{S,X}$.
- $L_{S,R_i}$ is L-interfered by $L_{S,R_j}$ if none of them is detectable when they coexist.

The L-interfered relation is symmetric while L-independent and L-dependent relations are not. At a given receiver node, when the reception threshold of $L_1$ is the same as that of $L_2$, Fig. 1: Examples of a three-link network. From (a), (b) is obtained by moving $L_1$ and $L_3$ to the right side and (c) is obtained by moving $L_3$ to the right side.
the fact that $L_3$ is $L$-dependent on $L_2$ implies that $L_2$ is $L$-independent of $L_1$.

**Property 1** At any given node, if the reception threshold is the same for all links, $L$-dependent relation is transferable.

*Proof:* Consider, at node $X$, $L_{S,R_1}$ is $L$-dependent on $L_{S,R_2}$ and $L_{S,R_3}$ is $L$-dependent on $L_{S,R_4}$. Let $\beta_X$ be the reception threshold at $X$, we have

$$\frac{P_X(S_3)}{P_{N_0} + P_X(S_3)} \geq \frac{P_X(S_3)}{P_{N_0} + P_X(S_2)}, \frac{P_X(S_2)}{P_{N_0} + P_X(S_1)} \geq \beta_X \cdot \beta_X \geq \beta_X.$$

So $L_{S,R_1}$ is $L$-dependent on $L_{S,R_2}$, which means that $L$-dependent is transferable.

### B. Network Interference Model

Whether a set of links can concurrently transmit or not is determined by the local interference at all intended receivers. Let $LL_L(X)$ denote the set of links by which link $L$ is $L$-interfered at node $X$ and $DL_L(X)$ the set of links on which link $L$ is $L$-dependent at node $X$. Link $L$ is *survivable* at node $X$ only when no link in $LL_L(X)$ is active and every link in $DL_L(X)$ is either not transmitting a signal or survivable at node $X$. A link set $LS$ is *feasible* only if every link is survivable at the intended receiver when all links in $LS$ are active.

Conflict graph [6] is a widely used method to model network interference. It takes every link as vertex, and connects two vertices if the corresponding links are interfered. In a CG, it only state whether two links can transmit concurrently, but does not provide any information to distinguish the two types of interference and differentiate the simultaneous transmission due to capture effect from that due to SIC. To model the link dependence and indirect interference, we introduce a new type of vertex, *super vertex*, which contains both the link and the correlated one(s). For example, if $L_{S,R_1}$ is $L$-dependent at $R_1$ on $L_{S,R_2}$, a super vertex $(L_{S,R_1},L_{S,R_2})$ is constructed to capture the dependence.

As the $L$-dependent relation may be transferable, there can exist a dependent chain, e.g., at $R_1$, $L_{S,R_2}$ depends on $L_{S,R_1}$, and $L_{S,R_3}$ depends on $L_{S,R_2}$, etc. Then a problem arises: How many links should a super vertex at least contain? The following claim helps us to answer the question.

**Claim 1** Under the proposed link interference model, a link set consisting of more than three links is feasible, if and only if all subsets of two links and of three links are feasible.

It is feasible to set a limit that there are only two links in a super vertex. With claim 1, no matter how large the network is, it is sufficient to model the relation of every subset of two or three links. When there are two links, direct interference is captured, and whether or not they can transmit simultaneously can be indicated by the absence or presence of edge between the two corresponding vertices in the network graph. Consider a three-link set, denoted by $\{L_1, L_2, L_3\}$. Without loss of generality, suppose $L_1$ depends on $L_2$, and $L_3$ interferes the detection on $L_2$ at the receiver of $L_1$. To capture the indirect interference, it is sufficient to model the dependence of $L_1$ on $L_2$ and the local interference of $L_3$ on $L_2$. The super vertex to describe the dependence contains only two links, i.e., $L_1$ and $L_2$.

*Remark:* Claim 1 indicates that a subset of three links is the minimum unit to characterize indirect interference. To capture the effect of SIC, it is not enough to model only the relation of two links. As a result, many available models cannot capture the important network features when SIC is applied.

### C. Simultaneity Graph: Construction

Graph is a common way to model wireless networks. We present a new model, *simultaneity graph* (SG). Let $SG=(V,E)$ denote a simultaneity graph, where $V$ collects all vertices and $E$ all edges. Table II summarizes the rules to construct a simultaneity graph.

<table>
<thead>
<tr>
<th>Rule</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>Vertex Rule I</td>
<td>There is an OV for every link, i.e., $(L)$ for link $L$</td>
</tr>
<tr>
<td>Vertex Rule II</td>
<td>There is an SV for an $L$-dependent pair, i.e., $(L_{S,R_1},L_{S,R_2})$ if $L_{S,R_1}$ is $L$-dependent on $L_{S,R_2}$ at node $R_1$.</td>
</tr>
<tr>
<td>Edge Rule I</td>
<td>Connect $(L_{S,R_1})$ to $(L_{S,R_2})$ if $L_{S,R_1}$ is $L$-interfered on $L_{S,R_2}$ by $L_2$.</td>
</tr>
<tr>
<td>Edge Rule II</td>
<td>Connect $(L_{S,R_1})$ to $(L_{S,R_2},L_{S,R_3})$ if $L_{S,R_1}$ is $L$-interfered on $L_{S,R_2}$ by $L_3$ at node $R_1$.</td>
</tr>
<tr>
<td>Edge Rule III</td>
<td>Connect $(L_{S,R_1})$ to $(L_{S,R_2})$ bidirectionally if $L_1=L_2$ or $L_1=L_3$.</td>
</tr>
</tbody>
</table>

There are two types of vertex in SG. An *ordinary vertex* (OV) contains a single link. For every link $L$, there is an OV $(L)$ in SG. A *super vertex* (SV) contains two links that are $L$-dependent. SV is order-aware. For example, there are two SVs in SG, $(L_{S,R_1},L_{S,R_2})$ and $(L_{S,R_2},L_{S,R_1})$, if $L_{S,R_1}$ is $L$-dependent on $R_1$ on $L_{S,R_2}$ and $L_{S,R_2}$ is $L$-dependent on $R_2$ on $L_{S,R_1}$.

There are three types of *directed* edges in SG. First, there is an edge from $(L_{S,R_1})$ to $(L_{S,R_2})$ if $L_{S,R_1}$ is $L$-interfered at $R_1$ by $L_{S,R_2}$. The connection between two OVs captures direct interference, the same as in a conflict graph. Second, there is an edge from $(L_{S,R_1})$ to $(L_{S,R_2},L_{S,R_3})$ if $L_{S,R_2}$ is $L$-interfered at $R_1$ by $L_3$. The connection between OV and SV characterizes the indirect interference. Finally, to capture the primary interference, two ordinary vertices are connected bidirectionally if the transmitters are the same or the transmitter of one link is the same as the receiver of the other.

Applying the rules in Table II, simultaneity graphs can be drawn for the scenarios in Fig. 1. We begin by constructing all vertices. By vertex rule I, there are three ordinary vertices in SG. By vertex rule II, two super vertices are constructed to capture the dependence of $L_1$ on $L_2$ and $L_3$ in scenarios (a) and (b). There is only one super vertex in (c). Moreover, observe that no interference exists in (a), while in both (b) and (c) indirect interference occurs. Therefore, edges between SV and OV are constructed according to edge rule II. There is also direct interference in (c), which is captured by the connection between the two ordinary vertices. The final results are shown in Fig. 2 (a)-(c). For comparison, the conflict graph for scenarios (a) and (b) is given in Fig. 2 (d).

From the simultaneity graphs in Fig. 2, it is clear that only in scenario (a) all three links are permitted to transmit concurrently, while in the other two scenarios simultaneous...
transmissions of two links are possible. In comparison, important network features cannot be captured in the conflict graph. As a result, it does not reveal the indirect interference in scenario (b) and incorrectly allows simultaneous transmission of all three links, which can result in the starvation of $L_3$.

Given $SV=(V,E)$, an adjacent relation is defined between two links or between a single link and a link set. A vertex is adjacent to another vertex if there is at least one edge between them. Note that the vertex can be an ordinary vertex or super vertex. Link $L$ is adjacent to link $L'$ when $(L')$ and $(L)$ are adjacent. Furthermore, link $L$ is adjacent to link set $LS$ if: (i) there is a link $x \in LS$ such that $x$ is adjacent to $L$; or (ii) there are two links $x, y \in LS$ such that $(xy)$ is adjacent to $(L)$; or (iii) there are two links $x, y \in LS$ such that $(xy)$ is adjacent to $(y)$ or $(Lx)$ is adjacent to $(y)$.

**D. Simultaneity Graph: Properties**

The remaining question is, based on a simultaneity graph, how to quickly determine the feasibility of any link set. The claims below show that, to determine the feasibility of a link set is equivalent to determine the connectivity of a specific vertex set. For link set $LS$, let $IVS(LS)$ denote the incident vertex set of $LS$, which contains all ordinary vertices $(L)$ if $L \in LS$ and all super vertices $(L_1L_2)$ if $L_1, L_2 \in LS$. Conversely, for vertex set $VS$, let $ILS(VS)$ denote the incident link set of $VS$, which contains all links $L$ if $L$ is in at least one vertex in $VS$ (i.e., there is a vertex such as $(L)$, $(Lx)$ or $(xL)$ in VS). A vertex set $VS$ is an independent set if every two vertices in VS are not adjacent, and a complete set if there is no vertex such that it is not in VS but only contains the link(s) in $ILS(VS)$.

**Property 2** Let $LS$ be a link set, then $IVS(LS)$ is a complete set. For vertex set $VS$, $VS = IVS(ILS(VS))$ if $VS$ is complete.

**Property 3** If link set $LS$ is feasible, then $IVS(LS)$ is an independent set.

When a vertex set is independent, it is not always true that the incident link set is feasible. To determine the feasibility of a link set from a vertex set, we have the following claims.

**Claim 2 (Soundness)** If vertex set $VS$ is an independent and complete set, then $ILS(VS)$ is feasible.

**Claim 3 (Completeness)** Link set $LS$ is feasible if and only if $IVS(LS)$ is a complete and independent set.

Proof: The sufficient condition is due to Property 2 and Claim 2, while the necessary condition is from Property 3.

Let $LS$ be a link set, Property 2 states the uniqueness of the incident vertex set for $LS$. Claim 3 further indicates that, to determine the feasibility of $LS$ is equivalent to determining whether $IVS(LS)$ is independent. From this aspect, $LS$ is represented uniquely by $IVS(LS)$ in a simultaneity graph.

**Algorithm 1**: Determine whether link $L$ can be added into feasible link set $LS$

**Data**: $SG (V,E)$: simultaneity graph; $LS$: a feasible link set; $L$: a link not in $LS$

**Result**: TRUE if $LS \cup \{L\}$ is feasible, FALSE otherwise

1. **foreach** vertex $e$ in $IVS(LS)$ **do**
   2. **if** $(L)$ is adjacent to $e$ **then**
   3. return FALSE;
   4. **end**
   5. **end**
   6. Let $SV_L \leftarrow V \cap \{(Lx),(xL)|x \in LS\}$;
   7. **foreach** link $e$ in $LS$ **do**
   8. **foreach** vertex $v$ in $SV_L$ **do**
   9. **if** $(e)$ is adjacent to $v$ **then**
   10. return FALSE;
   11. **end**
   12. **end**
   13. **end**
   14. return TRUE;

The most important task in scheduling is to find a feasible link set, which can be translated to find a complete and independent vertex set. Usually, as the scheduled link set is constructed gradually, we often need to decide whether a link (e.g., $L$) can be added into a feasible link set ($LS$). In a CG, for every $L' \in LS$, we need to check whether $(L)$ and $(L')$ are adjacent. In an SG, it is slightly more complex. Both direct and indirect interference should be examined carefully. The process is shown in Algorithm 1. The first **foreach** block serves to find all possible direct interference, which is the same as in the CG. Further, there are three classes of indirect interference. Namely, for $L_1, L_2 \in LS$,

- $L_1$ is interfered: At receiver of $L_1, L_1$ is $L$-dependent on $L_2$ and $L_2$ is $L$-interfered by $L_1$
- $L_1$ is interfered: At receiver of $L_1, L_1$ is $L$-dependent on $L$ and $L$ is $L$-interfered by $L_2$
- $L$ is interfered: At receiver of $L, L$ is $L$-dependent on $L_1$ and $L_1$ is $L$-interfered by $L_2$

The first class is checked in the first **foreach** block, while the last two classes are examined in the second **foreach** block. The time complexity of Algorithm 1 is polynomial. To determine the feasibility of $LS$, we should check the connectivity between every two vertices in $IVS(LS)$, i.e., between any two OVs and between any OV to any SV. Let $|LS|$ be the number of links in $LS$, as there are at most $O(|LS|^2)$ OVs and $O(|LS|^2)$ SVs, the total number of testing (i.e., line 2 and 9) is upper bounded by $O(|LS|^3)$.

**V. SCHEDULING POLICY**

Link scheduling has been studied extensively [5, 15]. However, most of them are not SIC-aware. We first analyze the link interference based on SG and then, for given link $L$, define...
interference number} to bound the number of links interfering or interfered by \( L \). The interference number plays a role in \( SG \) similar to that of the node degree in CG. As scheduling over \( SG \) is NP-hard, in the following, we present several heuristic solutions with theoretic performance bound.

We assume that the time is partitioned to slots of a constant duration. Each slot is for transmission of one packet. To measure the performance of a scheduling scheme, {schedule length} is defined as the total number of time slots used by the scheme. The objective of a scheduling policy is, given an \( SG = (V, E) \), to assign each link the required number of time slots while assuring the schedule length is as short as possible.

### A. Preliminary

As interference is the dominant factor to limit the number of simultaneous transmissions, in order to minimize the schedule length, it is critical to understand link interference. Previously, in CG, any link is contained by only one vertex and the degree of the vertex records the interaction of the link and the remaining part of the network. Hence, the design of many scheduling schemes over CG is based on node degree. However, in \( SG \), it is possible that a link is in more than one vertices, i.e., one ordinary vertex and several super vertices. Each vertex characterizes link interference from a unique aspect. Hence, to fully understand the interference of any given link, one should take all related vertices into account.

In particular, given \( SG=(V, E) \), let \( RVS(L) \) be relevant vertex set of \( L \), which denotes the set of all vertices containing link \( L \), i.e., \( RVS(L) = \{L\} \cup (V \cap \{Lx\}, (xL) x \in ILS(V)) \). The interference of \( L \) is specified completely by all vertices in \( RVS(L) \) together.

The interference number \( (IN) \) estimates the amount of transmissions affected by a link. Concretely, for link \( L \), both the links interfering and interfered by \( L \) should postpone their transmissions when link \( L \) is active. Given \( SG=(V, E) \), we define incoming number, \( \text{IN}^i_{SG}(L) \), as the number of links interfering \( L \), while outgoing number, \( \text{IN}^o_{SG}(L) \), as the number of links interfered by \( L \). Putting them together, the interference number of \( L \) is defined as \( \text{IN}_{SG}(L) = \text{IN}^i_{SG}(L) + \text{IN}^o_{SG}(L) \). The incoming number of link \( L \) can be determined as follows:

- For every \( y \in V \), if there is an edge from \( y \) to \( L \), increase \( \text{IN}^i_{SG}(L) \) by one;
- For every \( (Lx) \in E \) and every \( y \in V \), if there is an edge from \( y \) to \( (Lx) \) but no edge from \( y \) to \( (L) \), increase \( \text{IN}^i_{SG}(L) \) by one;
- If there are two edges connecting \( y \) to \( (Lx) \) and \( x \) to \( (Ly) \), decrease \( \text{IN}^i_{SG}(L) \) by one.

The first two items estimate the number of links that interfere \( L \) directly and indirectly, respectively. Consider two links, \( x \) and \( y \), which are \( L \)-interfered at the receiver of \( L \). If \( L \) depends on both of them, to ensure successful detection on \( L \), it is sufficient to postpone transmission on one of \( x \) and \( y \). Nevertheless, the second item counts twice for such indirect interference. To compensate for this, in the last item the incoming number is decreased by one.

Similarly, \( \text{IN}^o_{SG}(L) \) can be determined as follows:

- For every \( y \in V \), if there is an edge from \( (L) \) to \( y \), increase \( \text{IN}^o_{SG}(L) \) by one;
- For every \( (xL) \in V \) and every \( y \in V \), if there is an edge from \( (L) \) to \( (xL) \) but no edge from \( (y) \) to \( (x) \), increase \( \text{IN}^o_{SG}(L) \) by one.

There is no need for the outgoing number to count for the edge from \( (L) \) to \( (xy) \). Such edge corresponds to the fact that, at the receiver of \( x \), \( x \) depends on \( y \) and \( y \) is \( L \)-interfered by \( L \). The interference of \( L \) has already been counted. If \( x \) is \( L \)-interfered directly by \( L \), it is captured by the first item. Otherwise, if \( x \) is \( L \)-dependent on \( L \), the indirect interference introduced by \( L \) is captured by the second item. When \( L \) is active, either \( x \) or \( y \) should be silent to avoid indirect interference; yet, there is no need to postpone transmissions on both of them. As there is at most one link delayed by \( L \), we only increase the outgoing number by one in the second item.

**Property 4** Let \( SG = (V, E) \) be a simultaneity graph, for \( L \in ILS(V) \), the number of links postponed by \( L \) is upper bounded by \( \text{IN}_{SG}(L) \).

**Remark:** The interference number of a link \( L \) can be larger than the number of links delayed by \( L \) in practice. There may be redundancy between the incoming and outgoing numbers. Consider two links, \( L_1 \) and \( L_2 \), which are interfered by each other. Link \( L_2 \) is counted in both the incoming number and outgoing number of \( L_1 \). As a result, \( L_2 \) is counted twice in the interference number of \( L_1 \). By recording and comparing the links in the two categories (i.e., incoming and outgoing links), the redundance can be removed, which we leave as a future work.

For any link \( L \), the incoming number counts the links interfering \( L \). For direct interference, i.e., the edge from \( (y) \) to \( (L) \), it is captured in \( \text{IN}^o_{SG}(y) \). For indirect interference, i.e., the edge from \( (y) \) to \( (Lx) \), it is captured in \( \text{IN}^o_{SG}(x) \). Therefore, the sum of the outgoing numbers is no less than that of the incoming numbers.

**Property 5** Let \( SG = (V, E) \) be a simultaneity graph, then

\[
\sum_{LS \subseteq ILS(V)} \text{IN}^i_{SG}(L) \leq \sum_{LS \subseteq ILS(V)} \text{IN}^o_{SG}(L).
\]

Let \( LS \subseteq ILS(V) \), the subgraph of \( SG \) generated by \( LS \) is \( SG_{LS} = (V', E') \), where \( V' \) is the incident vertex set of \( LS \) in \( SG \) and \( E' \) collects all edges in \( E \) between any two vertices in \( V' \). The interference number of \( L \) to \( LS \) is defined as the interference number of \( L \) in \( SG_{LS}(L) \).

**Property 6** Let \( SG_1 = (V_1, E_1) \) be a simultaneity graph and \( SG_2 = (V_2, E_2) \) be a subgraph of \( SG_1 \). Then, \( \text{IN}_{SG_1}(L) \leq \text{IN}_{SG_2}(L) \forall L \in ILS(V_2) \). This also holds if replace \( IN \) by \( IN^i \) or \( IN^o \).

Given \( SG = (V, E) \), let \( \Delta^i(SG) \) denote the maximum interference number of \( SG \), i.e., \( \Delta^i(SG) = \max_{LS \subseteq ILS(V)} \text{IN}_{SG}(L) \). Similarly, \( \Delta(SG) \) and \( \Delta^o(SG) \) are the maximum incoming number and outgoing number of \( SG \), respectively. The following property is an immediate deduction of Property 6.

**Property 7** Let \( SG_1 = (V_1, E_1) \) be a simultaneity graph and \( SG_2 = (V_2, E_2) \) a subgraph of \( SG_1 \). Then, \( \Delta^i(SG_2) \leq \Delta^i(SG_1) \). This also holds if replace the superscript \( IN \) by
i or o.

SIC-aware scheduling over SG is at least NP-hard. It is clear that, after removing all super vertices and the incident edges, the SG reduces to the CG. That is, for a network where there is no need to apply SIC, the SG is the same as the CG. Scheduling over CG is known as NP-hard [6]. Hence, it is unlikely to design an optimal scheduling scheme over SG within a polynomial time. Otherwise, the same method can be applied to derive an optimal solution over CG. Next, three heuristic scheduling schemes are presented. The worst case performance bound of the maximal greedy scheme is shown. Besides the two independent set based schemes, we show that, with careful selection of link ordering, better theoretic bound can be achieved.

B. Independent Set based Policy

Though the scheduling over SG can be treated as vertex coloring of the subgraph induced by removing all super vertices, the node degree-based algorithms [5, 14] are no longer effective. In SG, the interference of a link \( L \) is characterized not only by the OV \((L)\), but by all the vertices in RVS\((L)\).

We study one class of schemes, referred to as independent set (IS) based schemes, where the interference number is used in place of node degree. A general process of an IS-based greedy scheme is given in Algorithm 2. Let \( S \) be the next scheduled link set (initially empty) to be constructed, \( U \) be the set of unscheduled links (initially containing all the currently unscheduled links) that can be placed in \( S \), and \( R \) the set (initially empty) of unscheduled links that cannot be placed in \( S \). The greedy scheme proceeds as follows:

- Choose the first link \( L \in U \) that has the maximum interference number. Place \( L \) in \( S \) and move all the links \( L' \in U \) that are adjacent to \( L \) from \( U \) to \( R \);
- While \( U \) remains nonempty, do the following: choose the first link \( L \in U \) with respect to a specific metric, add \( L \) to \( S \) and move all the links that are adjacent to \( L \) from \( U \) to \( R \).

When \( U \) has been emptied, \( S \) is exactly a link set that will be scheduled in a time slot. For a link requiring more than one slot, the procedure repeats several times until sufficient slots are assigned to the link. It is clear that Algorithm 2 computes a feasible schedule \( S_{IS} \).

The performance of an IS-based greedy scheme is determined by the way how the independent set (i.e., in steps 6-9) is generated. Two policies are considered in choosing the next link (step 8). The first policy, called Smallest Degree First (SDF), favors the link in \( U \) which minimizes the interference number to \( U \). The goal of the procedure is, by minimizing the number of links deleted from \( U \), to make current \( S \) as large as possible. The second policy, called Recursive Largest First (RLF), favors the link in \( U \) that has the maximum interference number to \( R \). Let \( G_R \) be the residual graph induced by the vertices except those in IVS\((S)\) after \( S \) is finally formed (i.e., when \( U \) is empty). The goal of the procedure is to make \( S \) large while assuring that \( G_R \) has as many edges eliminated from it as possible. In principle, RLF attempts to eliminate the interference at an early stage to enable more simultaneous transmissions for future. Concerning the average performance, RLF is usually the best among available graph coloring algorithms [16].

To clearly illustrate the difference of the two polices, see the example shown in Fig. 3, where each point represents a link. The three circles show the interference ranges of links \( L_1, L_2 \) and \( L_3 \), respectively. First, \( L_1 \) is chosen in step 4 of Algorithm 2. Afterwards, SDF and RLF make different decision in choosing the next link. After the links affected by \( L_1 \) are removed, as there are two links affected by \( L_2 \) and three by \( L_3 \), \( L_3 \) is preferred by SDF. In comparison, as there are two links affected by both \( L_1 \) and \( L_2 \) but none by both \( L_1 \) and \( L_3 \), \( L_2 \) is preferred by RLF.

The next claim states the worst case performance bound of a maximal greedy scheme. For simplicity, we assume that the required number of slots is at most one for every link. The case for demand larger than one is similar. A greedy scheme being maximal means that the set of scheduled links at any time slot except the last one is maximal, i.e., no more active link can be added. A link is active if it has packets to transmit (i.e., requiring at least one slot). It is easy to show that our IS-based greedy scheme is maximal.

Claim 4 Let \( SG=(V, E) \) be a simultaneity graph and \( T_X \) the schedule length of a greedy scheduling scheme \( X \). If \( X \) is maximal, then \( T_X \leq \Delta^{IV}(SG) + 1 \).

The role of interference number in SG is similar to that of
node degree in CG. It is well-known that the schedule length of maximal greedy scheme over CG is upper bounded by the maximum node degree. Claim 4 shows that the maximum interference number also bounds the schedule length for maximal greedy scheduling over SG.

C. The Third policy: Achieving Better Bound

It is known that, for graph coloring, the optimal coloring can be obtained by using a greedy approach on a certain ordering of vertices. To improve the general bound given in Claim 4, we present a new centralized scheduling scheme with a careful selection of link ordering.

Towards this, given \( \text{SG}=(V, E) \), let \( \text{IN}^d_{\text{SG}}(L) \) denote the difference of link \( L \), which is the difference between the outgoing and incoming IN, i.e., \( \text{IN}^d_{\text{SG}}(L) = \text{IN}^o_{\text{SG}}(L) - \text{IN}^i_{\text{SG}}(L) \).

**Property 8** Let \( \text{SG}=(V, E) \) be a simultaneity graph and \( \Delta^d(SG) \) the maximum IN difference of \( \text{SG} \), then \( \Delta^d(SG) \geq 0 \).

The scheduling scheme is summarized in Algorithm 3.

![Algorithm 3: SCHED - Scheduling with link ordering](image)

The following claim states the performance of Algorithm 3. For simplicity, we assume that the required number of slots is at most one for every link. The case for demand larger than one is similar.

Claim 5 Let \( SG=(V, E) \) be a simultaneity graph, the length of the schedule constructed by Algorithm 3 is upper bounded by \( O(\Delta^d(SG)) \).

Algorithm 3 shows that in-depth understanding of link interference can help us to achieve better scheduling performance in theory. Claim 4 bounds by \( \Delta^\text{IN}(SG) \) the schedule length of the maximal greedy scheduling scheme, while Claim 5 shows that Algorithm 3 needs at most \( 2 \cdot \Delta^\text{IN}(SG) + 1 \) slots. The new scheme improves the bound from \( \Delta^\text{IN}(SG) \) to \( \Delta^\text{d}(SG) \). It is likely that \( \Delta^\text{IN}(SG) \) is arbitrarily larger than \( \Delta^\text{d}(SG) \). Though in the worst case the performance bound is still up to \( O(n) \), the probability to have such a case can be low when the IN difference is exploited.

VI. SIMULATION RESULTS

We evaluate the performance of the scheduling schemes by simulations in NS-2 [17]. Each data point is obtained by averaging the results from multiple runs. Each run lasts until at least 1000 packets have been transmitted at every node.

**Simulation Setting:** Consider a single-channel network in a 500m \( \times \) 500m area. The two-way ground propagation model is used. Without interference, the transmission range is 250m. A TDMA protocol is deployed at the MAC layer. Transmission rate is fixed at 2Mbps for all nodes.

**SIC Support:** NS-2 provides support of capture effect. When collision occurs between two transmissions, the stronger one survives if the SINR is larger than CPThresh (a parameter in NS-2). We modify the function to support SIC and set CPThresh to the reception threshold. In particular, when collision occurs, for any involved signal, the reception succeeds only if (i) there is no L-interfered signal; (ii) the receptions of all L-dependent signals succeed; and (iii) the SINR is no less than CPThresh. Obviously, this check runs recursively.

**Traffic Load:** Unless otherwise specified, each node is backlogged, i.e., it always has packets to send, with packet size 1500 Bytes. For each source node, the destination is chosen randomly and a shortest multi-hop path is found.

Though there are several proposals to perform scheduling with MPR, the interference model used is not suitable for SIC. As the proposals such as those in [10, 11] heavily depend on model parameters, it is difficult to implement them in our scenarios. Similar to [4], we compare the performance of our schemes to that of IEEE 802.11. Note that IEEE 802.11 is a CSMA MAC protocol and not originally designed for MPR. We use it as reference to show the capability of SIC and the effectiveness of the proposed interference model and scheduling schemes. In the following, the results of the three proposed scheduling schemes are denoted by “LO”, “SDF” and “RLF”, respectively.

The performance metric is per-link throughput. At each receiver \( X \), both the packets destined to \( X \) and those forwarded by \( X \) are counted. Throughput gain is defined as \( (T_3 - T_{802})/T_{802} \), where \( T_3 \) is the throughput when scheme \( S \) is used and \( T_{802} \) is that when IEEE 802.11 with carrier sensing is deployed. As specified in NS-2, the carrier sensing range is 500m, i.e., twice of the transmission range.
A. Single-hop Networks

We first demonstrate the impact of SIC in the scenarios shown in Fig. 1. The results are plotted in Fig. 4, where subgraphs (a), (b) and (c) correspond to the three scenarios, respectively. The per-link throughput with TDMA scheduling is plotted with legend “Scheduling”. The schedules constructed by all the three schemes are the same. In addition, the results of IEEE 802.11 with and without carrier sensing are also shown. The use of carrier sensing prevents simultaneous transmission, i.e., the three links always transmit in sequence, though in (a) they can transmit concurrently. In opposite, when there is no carrier sensing, one can exploit the opportunity of simultaneous transmission, but there is no way to identify and avoid the collision that SIC fails to handle. When all the three links are active, in both (b) and (c), the receiver of link 1 is unable to extract the desired signal. As a result, link 1 is almost starved. With link scheduling, on one hand, a high throughput is obtained by allowing simultaneous transmission. In (a) all links transmit concurrently, while in (b) and (c) there are two links simultaneously transmitting in most slots. As compared to IEEE 802.11 with carrier sensing, the throughput gain is 180%, 100% and 50% in the three scenarios. On the other hand, nearly perfect fairness is achieved. Though link 1 suffers heavy interference, the service time is guaranteed by silencing link 2 or link 3 when link 1 is scheduled. The results indicate that SIC is capable to support simultaneous transmissions which are not permitted with single packet reception, while to exploit such capability, careful link scheduling is required.

B. Grid Networks

To investigate the potential advantage of SIC in a large network, we conduct simulation in a network with 49 nodes at uniform grids. Let \((x, y) (1 \leq x \leq 7, 1 \leq y \leq 7)\) denote the nodes at different positions. Three flow patterns are examined: (i) \(P1: (x, 1) \rightarrow (x, 7)\), (ii) \(X1: (x, 2) \rightarrow ((x+5) \mod 7, 6)\), and (iii) \(X2: (x, 7) \rightarrow ((x+5) \mod 7, 2)\). Here, \(v_1 \mod v_2\) return the the remainder obtained when dividing \(v_1\) by \(v_2\). There are seven multi-hop flows for each pattern. Fig. 5 shows the average per-link throughput gain, where “X1X2” refers to that all the flows of pattern X1 and X2 are active, and “PX” to that all flows are active.
Fig. 7: Average throughput gain verse the number of links in a network with 36 to 64 nodes.

It is clearly observed that a significant gain is obtained. The results imply that, with a larger number of active links, there are also more opportunities of simultaneous transmissions. Along with the increase of traffic load, a higher gain is achieved. Among the three schemes, “RLF” performs the best. When the load is heavy (e.g., X1X2 or PX), it almost doubles the throughput as compared to IEEE 802.11 with carrier sensing.

To analyze fairness, the cumulative distribution of per-link throughput is shown in Fig. 6 for pattern X1X2. The results of IEEE 802.11 are also plotted. The performance of IEEE 802.11 is very poor when carrier sensing is disabled. In fact, when the number of links is large, without any coordination, the interference is very high at most receivers. It is therefore not surprising that many links receive nearly zero throughput, and no link achieves relatively high performance. In comparison, with carrier sensing, though there are still some starved links, the link number is greatly reduced. Nevertheless, per-link throughput is very low, e.g., less than 40Kbps for almost 80% links. Carrier sensing also fails to ensure the fairness, i.e., there are a very small number of links receive a very high throughput. With link scheduling, no matter which scheme is used, the throughput is the same for more than 90% links. The results verify that, with SIC in place, the proposed schemes are able to achieve both high throughput and good fairness in a relatively large network.

C. Random Networks

We also carry out simulations of networks with dynamics. The number of nodes is varied between 36 to 64. All nodes are randomly positioned. For each node X, a transmission probability $p_X$ is assigned to determine whether it issues a packet or not at the beginning of a slot. In simulations, $p_X$ is the same for all nodes and varies from 0.5 to 0.9. Fig. 7 shows the average throughput gain verse the number of links. For “LO”, the gain is at least 30%. It is usually larger than 50% and up to 80%. The performance is even better for the two independent set based schemes. For example, when there are more than 30 links, the throughput is often double of or even higher than that of IEEE 802.11 with carrier sensing. Further, for all three schemes, it is expected that the performance gain increases with the number of links.

To further understand the performance gain, utilization ratio of super vertex is defined as the ratio of the number of used super vertices to that of all super vertices. A super vertex $(L_1, L_2)$ is used if both $L_1$ and $L_2$ are scheduled in the same slot when they are active. Fig. 8 shows the throughput gain verse the utilization ratio of SV. The correlation coefficient between them is given at row “Simulation” in Table III. The fact that all coefficient values are close to 0.5 indicates the essential correlation between the gain and the usage of super vertex. In general, with a larger utilization ratio, a higher gain can be expected. Moreover, there are much more super vertices used by “SDF” and “RLF”, which explains the additional gain of them as compared to that of “LO”. Though with a better theoretic bound, “LO” is not a superior choice. Recall that “LO” orders all links at once, while “SDF” and “RLF” only order the remaining links which are feasible with the set of scheduled links. It is interesting to study how to effectively make use of interference information, e.g., whether or not global ordering is a good option for a greedy scheme.

To capture the effect of SIC, super vertex is adopted to model link dependence. The complexity of scheduling relies on the number of super vertices in SG. Let $n$ denote the number of links in the network. The number of super vertices
through the number increases quickly, it is always bounded by the curve of function \( n \log n \). Note that in the worst case, the number can be up to \( O(n^2) \). Fig. 9 shows the number of super vertices verse the number of links. It is clear to see that the number can be up to \( O(n \log n) \). It is in fact bounded by \( O(n \log n) \). Note that in the worst case, the number can be up to \( O(n^2) \). Fig. 9 shows the number of super vertices verse the number of links. It is clear to see that the number can be up to \( O(n \log n) \). It is in fact bounded by \( O(n \log n) \).

In summary, the simultaneity graph accurately characterizes the impact of SIC with reasonable overhead. TDMA scheduling is capable to capture the opportunity of simultaneous transmissions and therefore provide significant performance gain over the classic CSMA protocol (e.g., IEEE 802.11).

VII. EXPERIMENTAL RESULTS

We now present preliminary experimental results on a software radio platform, the Universal Software Radio Peripheral (USRP) [18], built with Zigbee (i.e., IEEE 802.15.4 standard [19]) at the physical layer. We implemented a reference MAC and SIC, using a combination of standard and custom-built GNU radio processing blocks. For the details of implementation, please refer to [9] and the references therein. The IEEE 802.15.4 standard provides a physical layer for sensor networks and other wireless personal area networks. At 2.4GHz, it sends up to 128byte packets at a low rate of 250kbps. The true bitrate of the ZigBee PHY is 2Mbps. Low rate 802.11 modes share many features with ZigBee including comparable data rate and the use of DSSS.

The testbed is deployed in a large lab room with up to 11 USRP nodes randomly positioned. When there is no interference, every two nodes can communicate directly. Evidently, if all nodes do not perform SIC, all transmissions should take place in turn. Let \( U_i \) denote the \( i \)th node, where \( i \in \{1, 2, \ldots, 11\} \). A transmission probability is assigned to each node when it acts as a transmitter (e.g., \( p_i \) for \( U_i \)). For all nodes, \( p_i \) is the same and varies from 0.5 to 0.9. Two different patterns are examined.

- **C1**: \( U_1-U_{10} \) send packet to \( U_{11} \).
- **D1**: Every node is a transmitter and randomly chooses another node as the destination, which is changed randomly after every 100 packets are sent.

**Results**: For the networking scenarios, the results demonstrate clearly that SIC is a practical choice to provide high performance, and link scheduling is able to exploit the transmission opportunity. Each data point is obtained by averaging the results from multiple runs. Each run lasts for about 5 minutes.

**TABLE III**: Correlation coefficient between the throughput gain and the utilization ratio of super vertex

<table>
<thead>
<tr>
<th>#</th>
<th>LO</th>
<th>SDF</th>
<th>RLF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation</td>
<td>0.374</td>
<td>0.439</td>
<td>0.476</td>
</tr>
<tr>
<td>Experiment</td>
<td>0.612</td>
<td>0.616</td>
<td>0.553</td>
</tr>
</tbody>
</table>

The first pattern (C1) is similar to the infrastructure based WLAN. Among \( U_1, U_2, \ldots, U_{10} \), we randomly choose \( N \) (\( 5 \leq N \leq 10 \)) of them to be transmitters. Fig.10 plots the throughput gain verse the number of transmitters when \( p_i=0.9 \). Though slightly lower than that in simulations, a significant gain is obtained for most of the \( N \) values. In general, the gain is between 15% and 30%. Interestingly, the three schemes give similar performance. In some cases (e.g. \( N = 7 \) or 9), we found that, there is no need to apply SIC (i.e., every two signals are interfered at \( U_{11} \)). At this time, as all transmissions must be performed sequentially, there is no gain obtained from link scheduling.

Consider the second pattern (D1), which is similar to an ad hoc network. Due to the diversity of receiver, one can expect much more opportunities to apply SIC. The throughput gain verse the utilization ratio of SV for “SDF” is shown in Fig. 11. The results for “LO” and “RLF” are quite similar. Besides, the correlation coefficient for the three schemes is given at row “Experiment” in Table III. As compared to the pattern C1, the advantage is much more significant. For example, throughput gain is on average 40% and up to 80%. It is consistent with simulations that larger gain is achieved with a higher utilization ratio of SV. For example, when more than 30% super vertices are used, the throughput gain is usually larger than 40%. To investigate the overhead of SG, Fig. 12 shows the number of super vertices verse the number of links (i.e., \( n \)). It is the same as in simulations that the number of SV is always bounded by \( O(n \log n) \). We have observed that, in experiments, the utilization ratio often reaches or exceeds 20%, which is much higher than in simulations (Fig. 8). The results indicate that even when the network size is not large, SIC is able to provide many opportunities of simultaneous
transmission and, benefit from the potential MPR advantage, links should be coordinated carefully.

VIII. CONCLUSIONS AND FUTURE WORK

Though SIC is a simple way to perform multipacket reception, scheduling in ad hoc networks with SIC is nontrivial. The fact that the links detected sequentially by SIC are correlated at the receiver poses key technical challenges. We characterize the new link relation and propose simultaneity graph to capture the effect of SIC. We show that scheduling over SG is NP-hard and the maximum interference number bounds the performance of maximal greedy schemes. Three policies are explored to efficiently construct maximal feasible schedule. The performance is verified in both simulations using NS-2 and measurements in testbed. For future work, to combat the effect of both interference and attenuation, it is necessary to integrate interference cancellation and rate adaptation. Finally, an efficient distributed scheme to achieve good scheduling performance in a large-scale ad hoc network requires further investigation.

APPENDIX

Proof of Claim 1: The necessary condition holds intuitively. We proof the sufficient condition by contradiction. Let LS denote the whole link set. We need to show that, when LS is not feasible, there must be at least one infeasible subset of two or three links. If LS is not feasible, at least one link LS,R ∈ LS is not survivable. Then we have one (or both) of following:

- There is a link LS,R ∈ LS and LS,R ∈ LL̄LS(R);
- There is a link LS,R ∈ LS and LS,R ∈ DL̄LS(R); yet node R cannot decode the signal of LS,R.

For the first case, {LS,R, LS,R'} is not feasible. For the second case, similarly, we have:

- There is a link LS,R ∈ LS and LS,R ∈ LL̄LS(R);
- There is a link LS,R ∈ LS and LS,R ∈ DL̄LS(R); yet node R cannot decode the signal of LS,R.

For the first case, {LS,R, LS,R, LS,R'} is not feasible. The indirect interference of LS,R prevents the detection on LS,R.

For the second case, similarly, we have an LS,R or LS,R. The chain is extended until an LS,R is encountered for LS,R (e ≥ 1). As the number of links is finite, such e must exist.

As the reception threshold is larger than one, it is easy to see that P_R(S) ≤ P_R(S_1) . . . ≤ P_R(S_e). Therefore, LS,R must not be L-independent of L_{S,L,R}(k ≤ e). If LS,R is L-interfered by LS,R, {LS,R, LS,R} is not feasible. Otherwise, LS,R is L-dependent on LS,R, then {LS,R, LS,R, LS,R'} is not feasible because the indirect interference of LS,R prevents the detection on LS,R.

In summary, when the whole link set is not feasible, one can always find a two-link or three-link subset that is not feasible.

Proof of Property 3: We proof by contradiction, i.e., if IVS(LS) is not an independent set, LS must be infeasible. When IVS(LS) is not an independent set, IVS(LS) must contain:

- Two adjacent OVs, e.g., (L') and (L''); or
- an OV (L') and an SV (L''L) which are adjacent.

Then {L', L''} or {L_1, L_2, L_3} is not feasible. Observe that {L', L''} ⊆ LS and {L_1, L_2, L_3} ⊆ LS, according to claim 1, we can therefore conclude that LS is not feasible.

Proof of Claim 2: We proof by contradiction, i.e., if ILS(VS) is infeasible, VS is incomplete or not an independent set. According to Claim 1, there is a subset (e.g., Sub_1) of ILS(VS) that is not feasible and contains two or three links. When Sub_1 = {L_1', L_2'}, (L_1') and (L_2') must be adjacent. Otherwise, Sub becomes feasible. When Sub_2 = {L_1, L_2, L_3}, let L_1 be the failed link. In the presence of indirect interference, L_1 should be L-dependent on a link (e.g., L_2), which is L-interfered by the third link (e.g., L_3) at the receiver of L_1. As a result, (L_1, L_2) is constructed by the vertex rule II, which is adjacent to (L_3) according to the edge rule II. If VS is incomplete, we complete the proof. Otherwise, as {L_1', L_2'} ⊆ LS and {L_1, L_2, L_3} ⊆ ILS(VS), we have {L_1', L_2'} ⊆ VS or {L_1, L_2, L_3} ⊆ VS, implying that VS is not an independent set.

Proof of Claim 4: We proof by contradiction, i.e., if T_x > Δ^IN(SG) + 1, then the greedy scheme X is not maximal. Let LS_1, . . . , LS_n denote the schedule constructed by the scheme X. ∀L ∈ LS_T, we have IN_SG(L) ≥ Δ^IN(SG) < T_x-1. From the Pigeon hole principle, Among LS_1, . . . , LS_T-1, there must be at least one link set which does not contain any link postponed by L. Let LS denote the link set. It is safe to add L into LS. As L is active at time slot i (i.e., when LS_i is scheduled), the greedy scheme X is not maximal.

Proof of Property 8: Let L_1, . . . , L_n denote all the links in ILS(V).

\[
\Delta^d(SG) = \max_{1 ≤ S ≤ n} \left( |IN_{SG}^{n+1}(L_s) - IN_{SG}^n(L_s)| \right) \\
≥ \frac{1}{n} \sum_{1 ≤ S ≤ n} \left( IN_{SG}^n(L_s) - IN_{SG}^n(L_s) \right) \\
= \frac{1}{n} \left( \sum_{1 ≤ S ≤ n} IN_{SG}^n(L_s) - \sum_{1 ≤ S ≤ n} IN_{SG}^n(L_s) \right) \\
≥ 0.
\]

The last inequality holds due to Property 5.

Proof of Claim 5: Let n denote the total number of links and L_1, . . . , L_n the ordered links. Scheduling (i.e., steps 8-11) starts from L_n to L_1. After L_i is scheduled, let TS_i denote
the schedule length and $SG_t = (V_t, E_t)$ the simultaneity graph generated by the set of all scheduled links. It is obvious that $TS_1 \geq TS_2 \geq \ldots \geq TS_n$ and $TS_1$ equals to the length of the schedule constructed by Algorithm 3.

Observe that in $SG_t$, the IN difference of $L_t$ is the maximum one. According to Property 8, $\Delta(SG_t(L_t)) \geq 0$. That is, $\text{IN}^I_{SG_t}(L_t) \geq \text{IN}^O_{SG_t}(L_t)$. Thus,

$$\text{IN}_{SG_t}(L_t) = \text{IN}^I_{SG_t}(L_t) + \text{IN}^O_{SG_t}(L_t) \leq 2 \cdot \text{IN}^O_{SG_t}(L_t). \quad (8)$$

When we attempt to assign a time slot to $L_t$, there are two possible cases:

- $TS_{t+1} > \text{IN}_{SG_t}(L_t)$: Observe that the number of links postponed by $L_t$ is upper bounded by $\text{IN}_{SG_t}(L_t)$. From the Pigeon hole principle, there must be at least one time slot, for which the set of scheduled links does not contain any link postponed by $L_t$. It is safe to assign such slot to $L_t$. As a result, the schedule length is unchanged after $L_t$ is scheduled, i.e., $TS_t = TS_{t+1}$.

- $TS_{t+1} \leq \text{IN}_{SG_t}(L_t)$: In the worst case, one new time slot is allocated to $L_t$, implying $TS_t \leq TS_{t+1} + 1 \leq \text{IN}_{SG_t}(L_t) + 1 \leq 2 \cdot \text{IN}^O_{SG_t}(L_t) + 1$.

In summary, $TS_t \leq \max\{TS_{t+1}, 2 \cdot \text{IN}^O_{SG_t}(L_t) + 1\}$. Observe that $SG = SG_1$ and $SG_{t+1}$ is a subgraph of $SG_t$, we show $TS_t \leq 2 \cdot \Delta(SG_t) + 1$ via two steps.

(i) Evidently, $TS_n = 1 \leq 2 \cdot \Delta(SG_n) + 1$.

(ii) Suppose $TS_{k+1} \leq 2 \cdot \Delta(SG_{k+1}) + 1$ for $k \leq n$, then $TS_k \leq 2 \cdot \Delta(SG_k) + 1$ must hold. In fact,

$$TS_k \leq \max\{TS_{k+1}, 2 \cdot \text{IN}^O_{SG_t}(L_k) + 1\} \leq \max\{2 \cdot \Delta(SG_{k+1}) + 1, 2 \cdot \text{IN}^O_{SG_t}(L_k) + 1\} \leq \max\{2 \cdot \Delta(SG_k) + 1, 2 \cdot \text{IN}^O_{SG_t}(L_k) + 1\} = 2 \cdot \Delta(SG_k) + 1. \quad (9)$$

The third inequality holds due to Property 7.

Put the results in the two steps together, we conclude that $TS_t \leq 2 \cdot \Delta(SG_t) + 1$. Then $TS_1 \leq 2 \cdot \Delta(SG_1) + 1 = 2 \cdot \Delta(SG) + 1$. Therefore, the length of the schedule constructed by Algorithm 3 is upper bounded by $O(\Delta(SG))$. ■

REFERENCES


