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Frequency-Selective Time-Varying Downlink Scheduling and Resource Allocation of an LTE Cellular System

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Abstract

New adaptive Orthogonal Frequency Division Multiple Access (OFDMA) scheduling and resource allocation algorithms for frequency-selective time-varying downlink channels are proposed. Our adaptive strategies can be used in LTE-based systems, which are OFDMA-based in downlink and incorporate a shared-channel transmission scheme. In the proposed approach, system throughput, QoS constraints, and scheduling fairness are jointly integrated into a framework to dynamically perform radio resource-allocation for multiple users, and effectively choose optimal system parameters such as power and modulation rate to adapt to the varying channel quality of each resource block. In such systems, frequency-time resources are dynamically shared between users to enhance overall spectral efficiency. We obtain optimum power and continuous- and discrete-rate adaptation in order to maximize channel spectral efficiency, given a limited number of time-frequency resource blocks. In accordance with our analytical results, two different algorithms are then proposed for continuous and discrete resource scheduling. Simulation results showed significant performance enhancement by using the proposed system.
Index Terms

Frequency-Selective, Time-Varying, Scheduling, Resource Allocation, LTE, OFDMA, Continuous Rate Adaptation, Discrete-Rate Adaptation

I. INTRODUCTION

In order to make all users share system resources in broadband wireless communication, different multiple access methods such as FDMA, TDMA, and CDMA are used. As one of the frequency division techniques, Orthogonal Frequency Division Multiplexing (OFDM) keeps all sub-carriers overlapping and orthogonal, and has become a competitive technique in wireless communication for its spectrum efficiency and robustness against multi-path fading [1]. Orthogonal Frequency Division Multiple Access (OFDMA) inherits OFDMs immunity to inter-symbol interference and frequency-selective fading. Although signal corruption due to a frequency-selective channel can, in principle, be handled by equalization at the receiver side, the complexity of the equalization starts to become unattractively high for implementation in a mobile terminal at bandwidths above 5 MHz. Therefore, OFDM with its inherent robustness against frequency-selective fading is attractive for the downlink, especially when combined with spatial multiplexing. In addition, OFDM provides access to the frequency domain, thereby enabling an additional degree of freedom to the channel-dependent scheduler compared to High Speed Packet Access (HSPA). From a baseband perspective, flexible bandwidth allocation is easily supported by OFDM. This can be achieved by varying the number of OFDM sub-carriers used for transmission. The Long Term Evolution (LTE) downlink transmission scheme is based on OFDM technology [2].

At the heart of the LTE transmission scheme is the use of shared-channel transmission, in which time-frequency resources are dynamically shared amongst users. This is similar to the approach taken in High-Speed Downlink Packet Access (HSDPA), although the realization of the shared resource differs between the two: time and frequency in the case of LTE [3], [4], and time and channelization codes in the case of HSDPA [5], [6]. The use of time-frequency shared-channel transmission is well matched to the rapidly varying resource requirement posed by packet data and also enables several other key technologies.
used by LTE. The scheduler controls, for each instant of time, to which users the shared resources should be assigned. It also determines the data rate used for each link. However, LTE has, in addition to the time domain, access to the resources in frequency domain, due to the use of OFDM technology in the downlink. Therefore, the scheduler can, for each frequency region, select the user with the acceptable channel conditions. In other words, scheduling in LTE can take channel variation into account not only in the time domain, as with HSPA, but also in the frequency domain [7], [8]. Each user is assigned to a number of so-called resource blocks in the time-frequency grid, as illustrated in Fig. 1. In LTE this type of resource block assembles 12 subcarriers and has a bandwidth of 180 kHz. In the time domain, the resource block has a subframe duration of only 1 ms. Such a short subframe enables the exploitation of channel variations by scheduling users depending on their current channel quality. Concurrently, a short hybrid ARQ (HARQ) round-trip time of only 8 ms can be obtained. By allocating a variable number of resource blocks to a certain user and selecting a modulation and coding scheme to meet the current channel conditions, widely scalable transport block sizes are possible, resulting in a wide range of user-data rates [7], [9], [10].

Adaptive sub-carrier, bit, and power allocation in OFDM systems have been investigated [11]–[13], and scheduling algorithms were considered based on gradient-based methods, which select the transmission rate with the maximum projection onto the time-varying gradient of the system’s total utility [14]–[16]. Most of these works show that when channel state information (CSI) is available at the transmitter, system capacity can be greatly increased by exploiting the frequency domain as well as multi-user diversity. However, this type of resource-allocation method does not take the time-varying nature of a fading channel into account, especially when the channel is fast fading.

In this paper, we first address the optimum power and continuous rate adaptation for one resource block of an LTE-based system under total transmission power constraint for the base-station transmission. The results are extended to continuous rate adaptation in a multi-user case under power and fraction of time-frequency resource block constraints. The main goal of this paper is to obtain both optimum power
and continuous-rate in terms of time-varying and frequency-selective fading channel response. The other goal of this paper is to derive the respective optimum solutions for discrete-rate adaptation and assess the performance loss in the discrete-rate scenario compared with the continuous-rate case. This will be done by restricting the modulation rate to take on values from a finite set of integer rates.

The outline of this paper is as follows: In Section II, the system model and problem formulation is presented. In Section III, optimum power and rate adaptation are derived for continuous and discrete-rate adaptation cases. In Section IV, our proposed resource-allocation algorithms and simulation results are presented. The final section provides the conclusion of the work.

II. System Model and Problem Formulation

In this research, we consider a single-cell downlink scenario for an LTE cellular system using adaptive modulation (AM). Transmission power is assumed to be distributed among all sub-carriers. OFDMA is the multi-carrier modulation technique that has been adopted in the downlink of LTE cellular networks. The radio frame of LTE OFDMA is divided into \( I = 20 \) slots of 0.5 ms length in the time domain. Each time-slot normally carries seven symbols. As the basic time-frequency unit in the scheduler, a resource block consists of 2 time-slots and 12 adjacent sub-carriers. According to the LTE standard, each resource block can only be assigned to one user within a given cell [17]. Also, we assume all downlink resource blocks that are assigned to a single user can only adopt one modulation scheme within each scheduling period. Additionally, within each resource block, the quality of the sub-carriers may differ due to frequency selectivity of the channel. In order to avoid inter-symbol interference in a frequency-selective fading channel, the bandwidth of each sub-carrier is chosen to be sufficiently smaller than the coherence bandwidth of the channel. It is then reasonable to assume that the base-station transmits data over \( N \) parallel, independent sub-channels to mobile users, with individual QoS requirements [7]. We also assume that a discrete-time channel with stationary and ergodic time-varying gain \( g \) follows a given probability density function (pdf) \( p(g) \). In a block fading channel, the channel gain \( g_{ni} \) and frequency \( f_{ni} \) are constant over \((n, i)^{th}\) resource block, after which resource block \( g_{ni} \) changes to a new independent value.
based on the distribution \( p(g_{ni}) \), where \( p(g_{ni}) \) is pdf of stationary and ergodic time-varying channel gain \( g_{ni} \). The adaptive power allocated to the \((n, i)\)th resource block is shown by \( P_{ni}(g_{ni}) \) which should satisfy the power constraint at the base station: 
\[
\int_0^\infty P_{ni}(g_{ni})p(g_{ni}) \, dg_{ni} \leq P_{total}^{ni},
\]
where \( P_{total}^{ni} \) is the maximum total transmission power allowed over the \((n, i)\)th resource block. The downlink frequency-time domain OFDM frame structure, which will be examined in this work, is shown in Fig.1. A multi-user system with

![Fig. 1. The LTE downlink physical resource based on OFDM](image)

\( J \) users is also considered. The proportion of the time-frequency \((n, i)\)th resource block assigned to user \( j \in \{1, 2, \cdots, J\} \) is denoted by \( x_{j\,ni} \), \( x_{j\,ni} \leq 1 \). This part of this resource can only be allocated to at most one user. Moreover, this must satisfy \( \sum_{j=1}^J x_{j\,ni} \leq 1 \). In a multi-user system, where the channel gain of \( j \)th user over the \((n, i)\)th resource block is \( g_{j\,ni} \), the power allocated to this user is denoted by \( P_{j\,ni}(g_{j\,ni}) \). This must satisfy the total power constraint at the base station:
\[
\sum_{j=1}^J \int_0^\infty P_{j\,ni}(g_{j\,ni})p(g_{j\,ni}) \, dg_{j\,ni} \leq P_{total},
\]
where \( P_{total} \) is the maximum total transmission power.

We also consider a discrete-rate adaptation scheme, where a finite set of \( Z \) signal constellations is available. Here, we assume the same model as continuous rate adaptation policy but restrict the optimization problem. We assume a set of square constellations of size \( M_0 = 0, M_z = 2^{2(z-1)}, z = 1, \cdots, Z \) for some \( Z \). At each time-slot, we transmit symbols from a constellation in the set \( \{M_z : z = 0, 1, \cdots, Z\} \); the choice of constellation depends on the channel gain denoted by \( g_{ni} \) over the respective time-slot and frequency sub-carrier. Frequency boundaries in LTE downlink are shown in Fig.2.
III. OPTIMUM POWER AND RATE ADAPTATION

A. Continuous-Rate Adaptation / Single-User System

In each resource block, decision on scheduling and resource-allocation can be viewed as picking a rate $R_{ni} = (R_{11}, R_{21}, \cdots, R_{N1}, R_{N2}, \cdots, R_{NI})$, from the feasible rate region $\mathcal{R}_1(g_{ni}) \subset \mathbb{R}_{+}^N$, where $R_{ni}$ is an achievable data rate over the $(n,i)$th resource block. Denote $\overline{\gamma} = P_T/(N_T B)$ and $\overline{R}_{ni}$ as the average SNR over the whole frequency band ($B = N \times B_n$) and average achievable data rate over $(n,i)$th resource block, respectively. The bandwidth of the $n$th sub-carrier is denoted by $B_n$. $P_T$ and $N_T$ are average transmission power and noise power spectral density, respectively. When the channel is assumed to be Rayleigh fading, and assuming that M-ary adaptive QAM modulation is used for the proposed system, $\overline{R}_{ni}$ can be written as

$$\overline{R}_{ni} = \int_0^{\infty} B_n \cdot \log_2(1 + \alpha \cdot \gamma_{ni}(g_{ni})) p(\gamma_{ni}(g_{ni})) dg_{ni}. \quad (1)$$
where $\gamma_{ni}(g_{ni})$ is the received SNR corresponding to the $(n, i)^{th}$ resource block and can be expressed as [18]:

$$\gamma_{ni}(g_{ni}) = \frac{g_{ni} \times P_{ni}(g_{ni})}{B_n}, \quad (2)$$

and $\alpha$ is a constant for the user’s specific instantaneous Bit Error Rate (BER) requirement, which can be written as [18]:

$$\alpha = \frac{1.5}{(-\ln(5.\text{BER}))}. \quad (3)$$

For the feasible rate region, we focus on a model appropriate for downlink OFDM systems. Similar models have been considered in [8], [16]. The achievable rate region $\mathcal{R}_1(g_{ni})$ can be written as

$$\mathcal{R}_1(g_{ni}) = \left\{ R : \mathcal{R}_{ni} = \int_{0}^{\infty} B_n \cdot \log_2 \left(1 + \alpha \cdot \gamma_{ni}(g_{ni})\right) p(g_{ni}) \, dg_{ni}, \right.$$

$$\left. \int_{0}^{\infty} P_{ni}(g_{ni}) \cdot p(g_{ni}) \, dg_{ni} \leq P_{\text{total}}^{ni}, P_{ni}(\cdot) \in X_1 \right\}, \quad (4)$$

where

$$X_1 := \left\{ P_{ni} : 0 \leq P_{ni}(g_{ni}) \leq P_{\text{total}}^{ni} \quad \forall n, i \right\} \quad (5)$$

In this framework, we first maximize the data rate over each resource block according to the following constrained optimization problem

$$\max_{P_{ni}(g_{ni})} \mathcal{R}_{ni} = \max_{P_{ni}(g_{ni})} \int_{0}^{\infty} B_n \cdot \log_2 \left(1 + \frac{\alpha \cdot g_{ni} \times P_{ni}(g_{ni})}{B_n}\right) p(g_{ni}) \, dg_{ni},$$

subject to: $\int_{0}^{\infty} P_{ni}(g_{ni}) \cdot p(g_{ni}) \, dg_{ni} \leq P_{\text{total}}^{ni}, \quad (6)$
where \( P_{\text{ni}}^{\text{total}} \) is a positive constant. The optimal power allocation that maximizes (6) is found by using the Lagrangian function \( J(P_{ni}(g_{ni}), \lambda) \) given by

\[
J(P_{ni}(g_{ni}), \lambda) = \int_0^\infty B_n \cdot \log_2(1 + \alpha \cdot \frac{g_{ni} \times P_{ni}(g_{ni})}{B_n}) \cdot p(g_{ni}) \, dg_{ni} \\
- \lambda \left[ \int_0^\infty P_{ni}(g_{ni}) \cdot p(g_{ni}) \, dg_{ni} - P_{ni}^{\text{total}} \right],
\]

where \( \lambda \) is the Lagrangian multiplier. Optimizing with respect to \( P_{ni}(g_{ni}) \) given \( \lambda \) yields

\[
\frac{\partial J(P_{ni}(g_{ni}), \lambda)}{\partial P_{ni}(g_{ni})} = 0 \quad \Rightarrow \quad P_{ni}^*(g_{ni}) = B_n \cdot \left( \frac{1}{\lambda \cdot \ln(2)} - \frac{1}{\alpha \cdot g_{ni}} \right).
\]

Substituting this into optimization constraint in (6), we have

\[
\int_0^\infty B_n \cdot \left( \frac{1}{\lambda \cdot \ln(2)} - \frac{1}{\alpha \cdot g_{ni}} \right) \cdot p(g_{ni}) \, dg_{ni} \leq P_{ni}^{\text{total}},
\]

with Lagrangian multiplier, \( \lambda \), derived as

\[
\lambda^* = \frac{4 \alpha \cdot g_{ni}}{\left( \frac{\alpha \cdot g_{ni} \cdot P_{ni}^{\text{total}}}{B_n} \right)^2 - \alpha \cdot g_{ni} \cdot P_{ni}^{\text{total}} - 2}.
\]

Furthermore, channel gain in a time-varying frequency-selective fading channel can be expressed as

\[
g = |H(f, t)|^2,
\]

where \( H(f, t) \) is the channel transfer function and is the Fourier transform of time-varying channel impulse response \( h(\tau, t) \). Assuming the propagation channel consists of \( L \) discrete paths with different time delays, \( h(\tau, t) \) and \( H(f, t) \) can be expressed as [19]:

\[
h(\tau, t) = \sum_{l=0}^{L-1} h_l(t) \delta(\tau - \tau_l) \quad \text{and} \quad H(f, t) = \int_0^\infty h(\tau, t) e^{-j2\pi f \tau} \, d\tau,
\]

where \( h_l \) and \( \tau_l \) are complex channel gain and time delay of the \( l^{\text{th}} \) propagation path, respectively. Therefore, the optimal power allocation over time-frequency domain for the \((n, i)^{\text{th}}\) resource block,
\( P_{ni}^*(g_{ni}) \), is given by

\[
P_{ni}^*(g_{ni}) = P_{ni}^*(f, t) = B_n \left( \frac{1}{\lambda \ln(2)} - \frac{1}{\alpha |H(f, t)|^2} \right),
\]

(12)

and the corresponding Lagrangian multiplier, \( \lambda^* \), comes to

\[
\lambda^* = \frac{4\alpha |H(f, t)|^2}{\frac{(\alpha |H(f, t)|^2)^2 P_{total}^2}{B_n} - \alpha |H(f, t)|^2 P_{total} - 2}.
\]

(13)

Substituting \( P_{ni}^*(f, t) \) in (2), the maximum average data rate over the \((n, i)\)th resource block in given time-slot and frequency band yields to

\[
\bar{R}_{ni} = \int_0^\infty B_n \log_2 \left( \frac{\alpha |H(f, t)|^2}{\lambda \ln(2)} \right) p(H(f, t)^2) d(H(f, t)^2).
\]

(14)

where \( p(H(f, t)^2) \) is the probability density function (pdf) of \( H(f, t)^2 \). For Rayleigh fading channels, \( p(H(f, t)^2) \) follows exponential distribution.

**B. Continuous-Rate Adaptation / Multi-user Systems**

Since sub-channels and time-slots are independent, the total average data rate of the \( j \)th user over the entire frequency bandwidth \( B \) and \( I \) time-slots, is equal to the sum of average user data rates over all resource blocks and is given by

\[
\bar{R}_j = \sum_{n=1}^N \sum_{i=1}^I \bar{R}_{jni}.
\]

(15)

For a given allocation, \( j \)th user’s average rate over the \((n, i)\)th resource block is given by

\[
\bar{R}_{jni} = \int_0^\infty x_{jni} B_n \log_2 \left( 1 + \frac{\alpha g_{jni} P_{jni}(g_{jni})}{x_{jni} B_n} \right) p(g_{jni}) dg_{jni}.
\]

(16)

Therefore, over the multi-user system, the scheduling and resource-allocation decision can be viewed as selecting a rate \( R_j = (R_1, R_2, \cdots, R_J) \) from the feasible rate region \( \mathcal{R}_2(g_{ni}) \subset \mathbb{R}_+^J \). The optimal \( \bar{R}_j \) is
obtained by finding the optimal power allocation across resource blocks and is expressed by

\[
\mathcal{R}_2(g_{jni}) = \left\{ \sum_{j=1}^{J} \log_2(\bar{R}_j), \sum_{j=1}^{J} \sum_{n=1}^{N} \sum_{i=1}^{I} \int_{0}^{\infty} P_{jni}(g_{jni}).p(g_{jni}) \, dg_{jni} \leq P_{total}, \quad \text{and} \quad \sum_{j=1}^{J} x_{jni} \leq 1, \quad (x_{jni}, P_{jni}(\cdot)) \in \mathcal{X}_2 \right\},
\]

where

\[
\mathcal{X}_2 := \left\{ 0 \leq \sum_{j=1}^{J} \sum_{n=1}^{N} \sum_{i=1}^{I} P_{jni}(g_{jni}) \leq P_{total}, \quad 0 \leq x_{jni} \leq 1 \quad \forall j, n, i \right\}. \quad (17)
\]

In a multi-user scenario, fairness among users as well as QoS are important measures because they determine how smooth end-user applications are run on wireless systems. A well-known utility function used to strike a balance between system user capacity and fairness among \( J \) users is *proportional fairness utility function*:

\[
U_{PF} = \sum_{j=1}^{J} \log_2(\bar{R}_j). \quad (18)
\]

Due to concavity of \( \log_2(\cdot) \) function, there is a heavy penalty for situations with small \( \bar{R}_j \) (average data rate of \( j^{th} \) user). Therefore, the scheduler has to avoid the situation in which some users obtain very small throughput. This is because for the same throughput difference, the penalty of low-throughput terms is higher than the gain achieved by high-throughput terms [20]. Here, the scheduling and resource-allocation decision is based on selecting a rate from the feasible rate region \( \mathcal{R}_2(g_{ni}) \). Therefore, the optimization
problem for the proportional fair utility function (18) can be expressed as

$$\max_{P_{jni}(\cdot), x_{jni}} \sum_{j=1}^{J} \log_2(\bar{R}_j),$$

s.t. $$\sum_{j=1}^{J} \sum_{n=1}^{N} \sum_{i=1}^{I} \int_{0}^{\infty} P_{jni}(g_{jni}) \cdot p(g_{jni}) \, dg_{jni} \leq P_{\text{total}},$$

and $$\sum_{j=1}^{J} x_{jni} \leq 1 \quad \text{for} \quad n = 1, \ldots, N,$$

where $$P_{\text{total}}$$ is the total transmission power. Consider the Lagrangian given by

$$J(P_{jni}(g_{jni}), x_{jni}, \lambda, \mu_1, \mu_2, \cdots, \mu_N) =$$

$$\sum_{j=1}^{J} \log_2 \left[ \sum_{n=1}^{N} \sum_{i=1}^{I} x_{jni} \int_{0}^{\infty} B_n \cdot \log_2(1 + \alpha \cdot \frac{g_{jni} \times P_{jni}(g_{jni})}{x_{jni} \times B_n}) \cdot p(g_{jni}) \, dg_{jni} \right]$$

$$+ \lambda \left[ \sum_{j=1}^{J} \sum_{n=1}^{N} \sum_{i=1}^{I} \int_{0}^{\infty} P_{jni}(g_{jni}) \cdot p(g_{jni}) \, dg_{jni} - P_{\text{total}} \right]$$

$$+ \sum_{n=1}^{N} \mu_n (1 - \sum_{j=1}^{J} x_{jni}).$$

Optimizing over $$P_{jni}(g_{jni})$$ given $$x_{jni}, \lambda$$ and $$\mu_n, n = 1, 2, \cdots, N,$$ yields

$$\frac{\partial J(P_{jni}(g_{jni}), x_{jni}, \lambda, \mu_1, \mu_2, \cdots, \mu_N)}{\partial P_{jni}(g_{jni})} = 0 \Rightarrow P_{jni}^*(g_{jni}) = \frac{1}{W(\frac{\alpha g_{jni} \ln(2)}{x_{jni} B_n \lambda})} \cdot \frac{x_{jni} B_n}{\alpha g_{jni} \ln(2)},$$

where $$W(\cdot)$$ denotes the Lambert $$W$$-function which is the inverse function of $$f(W) = We^W$$. Substituting
this into \( J(P_{jni}(g_{jni}), x_{jni}, \lambda, \mu_1, \mu_2, \cdots, \mu_N) \), we have

\[
J(P_{jni}(g_{jni}), x_{jni}, \lambda, \mu_1, \mu_2, \cdots, \mu_N) =
\]

\[
\sum_{j=1}^{J} \log_2 \left( \sum_{n=1}^{N} \sum_{i=1}^{I} x_{jni} \int_{0}^{\infty} B_n \log_2 \left( 1 + \alpha \frac{g_{jni} \left( \frac{1}{x_{jni} B_n} \right)}{\alpha g_{jni} \ln(2)} \right) p(g_{jni}) dg_{jni} \right)
\]

\[
+ \lambda \left[ \sum_{j=1}^{J} \sum_{n=1}^{N} \sum_{i=1}^{I} \left( \int_{0}^{\infty} \frac{1}{W(\alpha g_{jni} \ln(2))} \frac{x_{jni} B_n}{\alpha g_{jni} \ln(2)} \right) p(g_{jni}) dg_{jni} - P_{\text{total}} \right]
\]

\[
+ \sum_{n=1}^{N} \mu_n (1 - \sum_{j=1}^{J} x_{jni}),
\]

and then optimizing this over \( x_{jni} \) given \( \lambda \) and \( \mu_n, n = 1, 2, \cdots, N \), yields

\[
\frac{\partial J(x_{jni}, \lambda, \mu_n)}{\partial x_{jni}} = 0 \quad \Rightarrow
\]

\[
\mu_n = \frac{1}{x_{jni}^*} - \alpha g_{jni} \left( \frac{1}{W(\alpha g_{jni} \ln(2))} \frac{x_{jni} B_n}{\alpha g_{jni} \ln(2)} \right)
\]

\[
\left( x_{jni}^2 B_n + \alpha g_{jni} \left( \frac{1}{W(\alpha g_{jni} \ln(2))} \frac{x_{jni} B_n}{\alpha g_{jni} \ln(2)} \right) \right) \log_2 \left( 1 + \alpha g_{jni} \left( \frac{1}{W(\alpha g_{jni} \ln(2))} \frac{x_{jni} B_n}{\alpha g_{jni} \ln(2)} \right) \right),
\]

\[
(22)
\]

Therefore, the optimal value of \( x_{jni}, x_{jni}^* \), with respect to the values of \( \mu_n, n = 1, 2, \cdots, N \), can be calculated by solving (23). For simplicity, we first find \( x_{jni}^* \) for all \( J \) users in the first sub-carrier and then extend over all frequency sub-carriers. This is shown in algorithm-1 in sub-section IV-A.

C. Discrete-Rate Adaptation

We determine the constellation size associated with each channel gain, \( g_{ni} \), by discretizing the range of channel gain and frequency levels. Specifically, we divide the range of channel gain \( g \) into \( \Phi \) and the range of frequency bandwidth \( B \) into \( \Psi \) regions: \( R_z = \{(g_{ni}, f_{ni}) | g_{ni} \in [g_{q-1}, g_q), f_{ni} \in [f_{s-1}, f_s)\} \), \( z = s, q = 0, \cdots, \Phi - 1 \), and \( s = 0, \cdots, \Psi - 1 \) where \( g_{-1} = 0, f_{-1} = 0 \) and \( g_{\Phi-1} = \infty, f_{\Psi-1} = B \). Hence,
transmission rate over \((g_{ni}, f_{ni}) \in R_z\) is \(k_z = \log_2 M_z\) bps/Hz for \(z > 0\) when the constellation size is \(M_z\). The adaptive MQAM design requires that the boundaries of the \(R_z\) regions be determined. These boundaries can be optimized to maximize spectral efficiency. Assuming Nyquist data pulses \((B = 1/T_s)\), where \(T_s\) denotes symbol rate, the achievable average rate over all the resource blocks is expressed by

\[
R_z(g_f, f_s) = \left\{ R_z = \sum_{n=1}^{N} \sum_{i=1}^{I} \sum_{q=0}^{\Phi-1} \sum_{s=0}^{\Psi-1} (f_s - f_{s-1}) k_z \int_{g_q}^{g_{q+1}} p(g_{ni}) dg_{ni}, \right. \\
\left. \sum_{n=1}^{N} \sum_{i=1}^{I} \sum_{q=0}^{\Phi-1} \sum_{s=0}^{\Psi-1} \int_{g_q}^{g_{q+1}} P_{qs}(g_{ni}) p(g_{ni}) dg_{ni} \leq P_{\text{total}}, \right. \\
\left. [f_{s-1}, f_s) \leq B_c, \quad \text{and} \quad (P_{qs}, f_s) \in X_3 \right\},
\]

where \(B_c\) is the coherence bandwidth and \(X_3\) is defined as

\[
X_3 := \left\{ 0 \leq \sum_{n=1}^{N} \sum_{i=1}^{I} \sum_{q=0}^{\Phi-1} \sum_{s=0}^{\Psi-1} P_{qs}(g_{ni}) \leq P_{\text{total}}, 0 \leq (f_s - f_{s-1}) \leq B_c \quad \forall q, s \right\}.
\]

The optimum rate \(M_z\) and the optimum power \(P_{qs}(g_{ni})\) in the discrete-rate version of the optimization problem can be obtained according to

\[
\max_{g_f} R_z = \max_{g_f} \sum_{n=1}^{N} \sum_{i=1}^{I} \sum_{q=0}^{\Phi-1} \sum_{s=0}^{\Psi-1} (f_s - f_{s-1}) k_z \int_{g_q}^{g_{q+1}} p(g_{ni}) dg_{ni}, \\
\sum_{n=1}^{N} \sum_{i=1}^{I} \sum_{q=0}^{\Phi-1} \sum_{s=0}^{\Psi-1} \int_{g_q}^{g_{q+1}} P_{qs}(g_{ni}) p(g_{ni}) dg_{ni} \leq P_{\text{total}}, \\
[f_{s-1}, f_s) \leq B_c, \quad \sum_{s=0}^{\Psi-1} (f_s - f_{s-1}) = B.
\]

Moreover, scheduling and resource-allocation take into account that choice of constellation size depends on the channel gain, \(g_{ni}\), over time and frequency bandwidth \((B_n)\). Therefore, by considering \(\alpha\) from (3), \(M_z = 1 + \alpha\gamma(g_{ni}) P_{qs}(g_{ni})\), the following relation between power and discrete-rate can be found

\[
P_{qs}(g_{ni}) = \frac{M_z - 1}{\alpha \gamma(g_{ni})} \quad g_q \leq g_{ni} \leq g_{q+1}.
\]
The Lagrangian for the respective optimization problem is

\[
L (g_q, f_s, \lambda, \mu_0, \mu_1, \cdots, \mu_{\Psi}) = \sum_{n=1}^{N} \sum_{i=1}^{I} \sum_{q=0}^{\Phi-1} \sum_{s=0}^{\Psi-1} f_s k_z \int_{g_q}^{g_{q+1}} p(g_{ni}) dg_{ni} \\
+ \lambda \left( \sum_{n=1}^{N} \sum_{i=1}^{I} \sum_{q=0}^{\Phi-1} \sum_{s=0}^{\Psi-1} \int_{g_q}^{g_{q+1}} P_{qs}(g_{ni}) p(g_{ni}) dg_{ni} - P_{total} \right) \\
+ \sum_{s=0}^{\Psi-1} \mu_s \left( (f_s - f_{s-1}) - B_c \right),
\]  

(28)

where \( \lambda \) and \( \mu_s, s = 0, 1, \cdots, \Psi - 1 \), are the Lagrangian multipliers and are set to satisfy constraints.

To find the optimum boundaries, we set the partial derivatives of the Lagrangian equal to zero

\[
\frac{\partial L (g_q, f_s, \lambda, \mu_0, \mu_1, \cdots, \mu_{\Psi})}{\partial f_s} = 0 \Rightarrow g_{q+1}^* = -\ln \left( \frac{\mu_s}{k_z} + e^{-g_q} \right),
\]  

(29)

for \( q = 0, 1, \ldots, \Phi - 1 \) and \( s = 0, 1, \ldots, \Psi - 1 \). This shows that for a given channel gain boundary \( (g_q) \), we can find the value of the next channel gain boundary \( g_{q+1} \). Moreover, frequency boundaries for a given channel gain \( (g_q) \) are obtained by

\[
\frac{\partial L (g_q, f_s, \lambda, \mu_0, \mu_1, \cdots, \mu_{\Psi})}{\partial g_q} = 0 \Rightarrow f_s^* = \left( \frac{\lambda}{k_z} \right)^{3/2} \left( \frac{M_z - 1}{\alpha g_{q+1} N_T} \right)^{3/4},
\]  

(30)

for \( n = 1, \ldots, N - 1 \) and \( i = 1, \ldots, I - 1 \).

The Lagrangian multipliers, \( \lambda \) and \( \mu_s, s = 0, 1, \cdots, \Psi - 1 \), in (29) and (30) are chosen so that average power and coherence bandwidth constraints of optimization problem are fulfilled, i.e.

\[
\sum_{n=1}^{N} \sum_{i=1}^{I} \sum_{q=0}^{\Phi-1} \sum_{s=0}^{\Psi-1} \int_{g_q}^{g_{q+1}} P_{qs}(g_{ni}) p(g_{ni}) dg_{ni} \leq P_{total} \quad \text{and} \quad (f_s^* - f_{s-1}^*) \leq B_c.
\]  

(31)

For each value of \( g_{ni} \) and \( f_{ni} \), we need to decide on the constellation size of the transmitted symbols and the associated transmit power. The rate at which the transmitter must change its constellation and power is obtained by algorithm-2 in sub-section IV-B.
IV. PROPOSED ALGORITHMS AND PERFORMANCE EVALUATION

A. Adaptive Continuous Rate Resource-Allocation

The target of the optimization algorithm is to provide sub-carrier assignment and power allocation for each user based on equations introduced in sub-section III-B. The algorithm starts by allocating a proportion \( x_{jni} \) of the first sub-carrier \( n = 1 \) in time-slot \( i = 1 \) to \( J \) users. This then continues for all sub-carriers within the same time-slot. When a proportion of sub-carriers in the \( i^{th} \) time-slot is allocated to each user, this process iterates over the remaining time-slots. It can be shown that the algorithm will stop in at most \( N \) (number of frequency sub-carriers) steps in each time-slot and in \( N \times I \) steps over all grids.

**Algorithm 1**: Joint scheduling and adaptive continuous-rate resource-allocation algorithm

\[
\begin{align*}
\text{for } n & \leftarrow 1 \text{ to } N \text{ do} \\
& \text{for } i \leftarrow 1 \text{ to } I \text{ do} \\
& \quad \text{for } j \leftarrow 1 \text{ to } J \text{ do} \\
& \quad \quad x_{jni} = \text{Eq. 23}; \quad \rightarrow x_{jni}(\mu_n, \lambda) \\
& \quad \quad \text{Solve } \sum_{j=1}^{J} x_{jni}^{*} - 1 = 0; \quad \rightarrow \mu_n^{*}(\lambda); \\
& \quad \quad \text{Substitute } \mu_n^{*}(\lambda) \text{ into } x_{jni}^{*}(\mu_n^{*}, \lambda); \quad \rightarrow x_{jni}^{*}(\lambda); \\
& \quad \quad \text{Substitute } x_{jni}^{*}(\lambda) \text{ into Eq. 21}; \quad \rightarrow P_{jni}^{*}(\lambda) \\
& \quad \quad \text{Solve } \sum_{j=1}^{J} \sum_{n=1}^{N} \sum_{i=1}^{I} \int_{0}^{\infty} P_{jni}^{*}(\lambda)p(g_{jni}) \, dg_{jni} - P_{\text{total}} = 0;
\end{align*}
\]

B. Channel- and Frequency-dependent discrete-rate adaptation

The proposed joint scheduling and resource-allocation algorithm over time-frequency domain in a Rayleigh fading channel for discrete-rate adaptation is given in algorithm-2. This algorithm, first calculates frequency and channel gain boundaries. Then, for each value of \( g_{ni} \) and \( f_{ni} \) of each resource block, the algorithm decides as to which constellation to transmit and what the associated transmit power should be. Channel gain regions, \([g_q, g_{q+1}]\), and frequency regions, \([f_s, f_{s+1}]\) for this algorithm are found from equations (29) and (30), respectively. As long as the values for \( g_{ni} \) and \( f_{ni} \) fit in these two regions, the relevant constellation size is allocated to the \((n, i)^{th}\) resource block.
Algorithm 2: Discrete-Rate Adaptation

Initialization: $f_0 = 0, q_0 = 0$

for $q ← 0$ to 7 do
  
  $g_{q+1}^* = -\ln \left( \frac{\mu_z}{e^{-g_q}} \right)$

  for $s ← 0$ to 7 do
    
    $f_s^* = \left( \frac{\lambda}{k_z} \right)^{3/2} \left( \frac{M_s-1}{\alpha_{q+1-N_T}} \right)^{3/4}$

    $g_{q+1}^* →$ Lookup Table

    $f_s^* →$ Lookup Table

for $n ← 1$ to $N$ do

  for $i ← 1$ to $I$ do

    for $q ← 0$ to 7 do

      if $g_q ≤ g_{ni} < g_{q+1}$ then

        for $s ← 0$ to 7 do

          if $s == 0$ then

            $M_0 = 0$

          else

            $z = s$

            $M_z = 2^{2(s-1)}$

C. Performance Evaluation and Simulation Results

In this sub-section, we present simulation results to illustrate the performance of our proposed resource-allocation schemes. The downlink of the OFDMA LTE system based on frequency band of $B = 5 MHz$ and $N = 256$ sub-carriers is simulated. According to the LTE standard [21], quadrature phase shift keying (QPSK), 16-quadrature amplitude modulation (QAM), and 64-QAM are supported on the downlink DSCH. Thus, this paper considers the AM schemes of QPSK, 16-QAM, and 64-QAM. The radio frame of OFDMA is divided into $I = 20$ slots of 0.5 ms length. Each time-slot carries seven symbols. As the basic time-frequency unit in the scheduler, a resource block consists of 2 time-slots and 12 adjacent sub-carriers. The spacing between sub-carriers $\Delta f$ is $15 kHz$. In Fig.3, maximum spectral efficiency resulting from optimized adaptive continuous rate MQAM under power constraint and fixed BER$= 10^{-3}$ is compared with channel capacity of the corresponding distribution derived in [18]. In Fig. 4, maximum spectral
efficiency of \( j = 3^{rd} \) user, \( (R_3/B) \) in a multi-users scenario with a fixed number of users, \( (J = 12) \) is plotted versus average SNR for \( BER = 10^{-3} \) and \( BER = 10^{-4} \).

Fig. 3. Maximum spectral efficiency for \( BER = 10^{-3} \), optimum rate adaptation in \((n, i)^{th}\) resource blocks

In Fig. 5, the \( U_{PF} \) in (18) is plotted versus average SNR values based on different BER constraints. Fig. 6, illustrates the impact of constellation restriction on adaptive MQAM. The power associated with each fading region for discrete-rate policy was chosen to guarantee an average \( BER = 10^{-3} \). It is clear that there is a small performance loss in the discrete-rate scenario compared with the continuous-rate case.

In Fig. 7 the optimum power adaptation \( , P(g) \), is plotted for discrete-rate case as a function of channel
Fig. 5. \( U_{PF} = \sum_{j=1}^{J} \log_2(\mathcal{R}_j) \) for BER = 10\(^{-3}\) and 10\(^{-4}\)

Fig. 6. Maximum spectral efficiency for BER = 10\(^{-3}\)

Fig. 7. \( P(g)/P_{\text{total}} \) for discrete-rate adaptation
gain $g$ where $BER = 10^{-4}$.

V. CONCLUSION

In this paper new adaptive OFDMA scheduling and resource-allocation algorithms for frequency-selective time-varying downlink channels were proposed for transmission over LTE cellular networks, in which channel spectral efficiency, application QoS, and scheduling fairness have been jointly considered to perform radio resource-allocation for multiple users. In the proposed system frequency-time resources are dynamically shared between users to enhance overall spectral efficiency. Based on the allocation results, the optimum power and continuous- and discrete-rate adaptation in order to maximize channel spectral efficiency were then obtained. The experimental results show that the proposed system results in significant performance improvement.

REFERENCES


